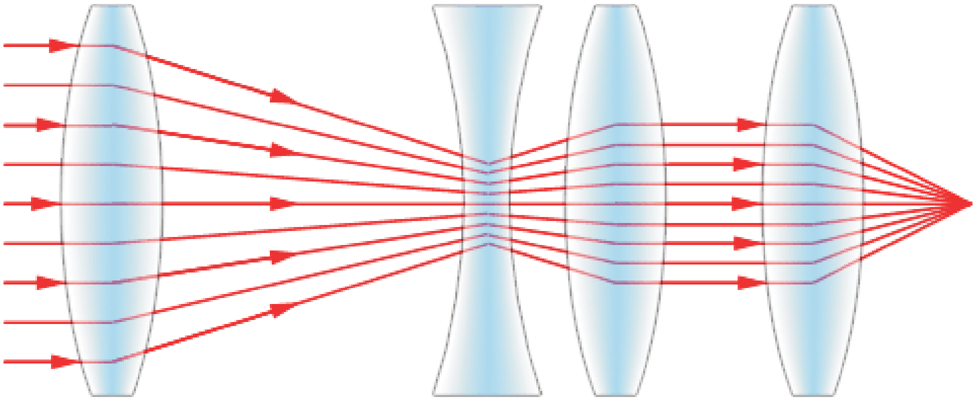
Thin Lenses and Optical Instrumentation



1 Purpose of the exercise

- To observe the operation of thin lenses and gain experience with placing and aligning optical components.
- To examine and measure real and virtual images in simple optical systems.
- To measure the focal lengths of double convex and double concave lenses.
- To understand the operation of simple optical instruments and to construct a simple microscope and telescope.

2 Theory

The goal of this exercise is to gain an understanding of the operation of simple optical instruments and construct a simple microscope and telescope. A general understanding of lenses as converging or diverging is therefore needed. We will in the following section introduce some useful equations and methods.

2.1 Standard equations

2.1.1 The thin-lens equation

For a thin lens it can be shown that the following equation is valid:

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}, \tag{1}$$

where s_o and s_i are the object and image distances, respectively, and f is the focal length of the thin lens (see figure 1).

2.1.2 The magnification factor

For an object height, y_o, and an image height, y_i, the magnification is given by

$$|M| = |\frac{y_i}{y_o}| = |\frac{s_i}{s_o}|, \tag{2}$$

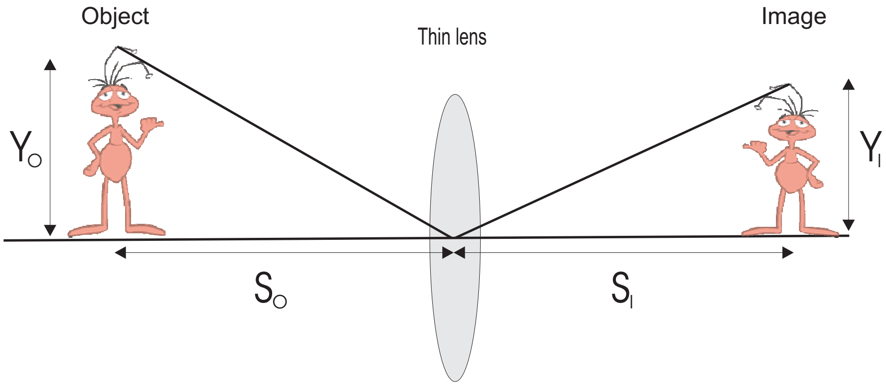


Figure 1: Thin lens.

2.2 Displacement method

A method useful for determining the focal length of a lens is the *displacement method*. It goes as follow. Suppose an object and a screen are located a fixed distance, b, apart (see figure 2). The converging lens whose focal length is to be measured is placed in between. As the lens is moved to different locations between the object and the screen it is found that a real image is focused on the screen only when the lens is in two specific positions. At one position the image is larger than the object and at the other position the image is smaller than the object. Define the distance a to be the separation between the two positions of the lens. Once a and b are know, the focal length can be found by the formula

$$f = \frac{b^2 - a^2}{4b} \tag{3}$$

Exercise:
Derive this equation. What is the longest focal length that can be measured for a given b*?*

One advantage of this method is that it permits the measurement of sizes and positions of inaccessible objects such as the filament inside a bulb or a virtual image located behind a lens. This will be used in the experiment to measure the position and magnification of a virtual image generated by a diverging lens.

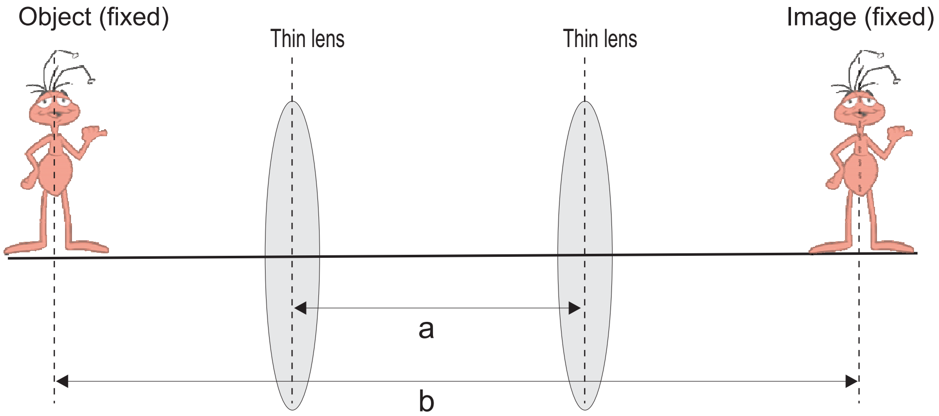


Figure 2: Displacement method.

2.2.1 Virtual image

By replacing the object in figure 2 with some sort of lens system that produces a virtual image, such as a concave lens (diverging lens). Then applying the displacement method with a lens of known focal length f one can find a value for a. Solving equation 3 gives then a value for b, the position of the virtual image. The size of the virtual image can also be measured. Suppose the virtual image has size y. Let y_1 and y_2 be the sizes of the real images on the screen for the two possible positions of the lens and let M_1 and M_2 be the corresponding magnifications. One can find:

$$1 = M_1 M_2 = \frac{y_1 y_2}{y^2} \tag{4}$$

The virtual image size is therefore

$$y = \sqrt{y_1 y_2} \tag{5}$$

Exercise:
Derive $M_1 M_2 = 1$.

2.3 Optical instruments

An optical instrument is a combination of optical elements that creates a magnified image of a small or distant object.

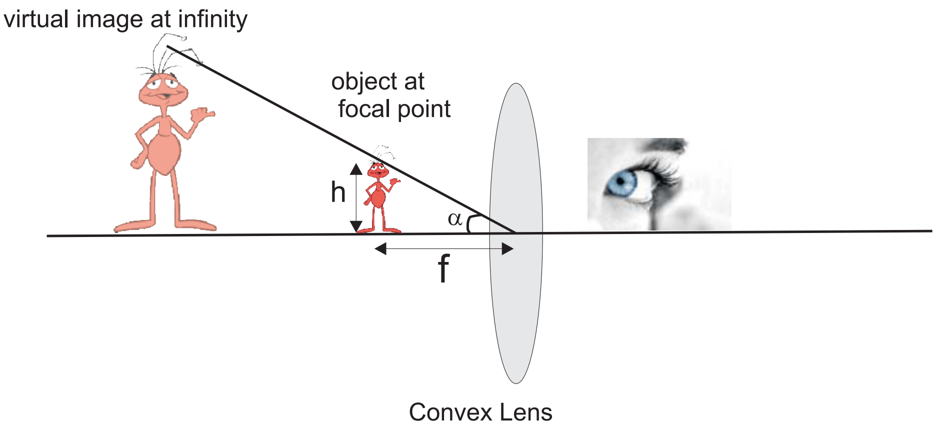


Figure 3: Magnifier.

2.3.1 Magnifying glass

One of the simplest optical instruments is the magnifying glass or eyepiece, which is a single converging lens which produces a virtual, magnified image of an object placed on or just inside the focal range of the lens. A magnifying glass works by creating a magnified virtual image of an object behind the lens. *The distance between the lens and the object must be shorter than the focal length of the lens for this to occur.* Otherwise, the image appears smaller and inverted, and can be used to project images onto surfaces. To understand the function of a magnifying glass, we first analyze a naked eye observer who is viewing a small object. Such an observer will try to make an object appear larger by bringing it closer to his or her eye. At a certain point, however, the object will become blurry as the eye can no longer accommodate the strong focus required to cast an image on the retina. This point is called the near point and is defined to be 25 cm (although of course the exact position at which this occurs will vary from observer to observer). The object at the near point observed with a naked eye subtends an angle (figure 3) α_0=arctan $h_0/25cm$. If we observe an object through a magnifying glass, we are in fact looking at its virtual, magnified image created near infinity (recall that for optical purposes, infinity means at a distance much larger than the scale of the experiment). The angular size of the image is the same as that of the object: $\tan\alpha = h_0/f$ (figure 3). However, because the image is far away, the eye will have no problem focusing on it. In this way, if $f < 25$ cm, the magnifying glass allows us to increase the angular size of the image by letting us bring the object closer to the eye. The angular magnification of the magnifying glass is given by the ratio of the apparent image's angle α to that of the angle α_0 made by the object when viewed by the unaided eye:

$$M_a = \frac{\tan \alpha}{\tan \alpha_0} = \frac{h_0/f}{h_0/25cm} = 25cm/f. \tag{6}$$

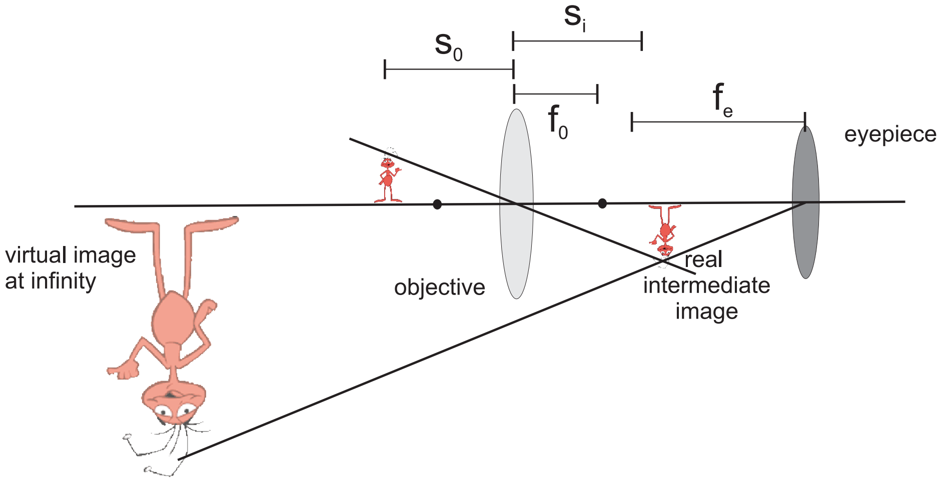


Figure 4: A simple compound microscope.

2.3.2 Compound microscope

Despite the advantage of being simple, the magnifying power of the simple magnifier is limited by aberrations. In order to achieve high magnification of a nearby object with fewer aberrations, the compound microscope is used, achieving magnifications much greater than that of the simple magnifier. A simple compound microscope is shown in figure 4. The objective lens produces a real, inverted, and magnified image known as the intermediate image. A simple magnifier is then used to produce a magnified virtual image of the intermediate image. The total magnification is given by the product of the magnification of the two lenses:

$$M = \frac{s_i 25cm}{s_0 f}. \tag{7}$$

2.3.3 The Telescope

In contrast to the microscope which magnifies the image of a near-by object with a generally high powered objective, the purpose of the telescope is to increase the retinal image of a distant object. Figure 5 displays the principles of a Keplarian telescope. The object is assumed to be at infinity so that incoming rays are parallel to one another. The objective lens forms a real, inverted intermediate image. If $s \rightarrow \infty$, $s_i \approx f$, so the image is formed at the focal point of the objective. The eyepiece once again produces, at infinity, a virtual image of the intermediate image. The magnification of the telescope is

$$M \approx \frac{\alpha}{\alpha_0} = -\frac{f_0}{f_e} \tag{8}$$

where the angles α, α_0 and focal lengths f_0, f_e are defined in figure 6.

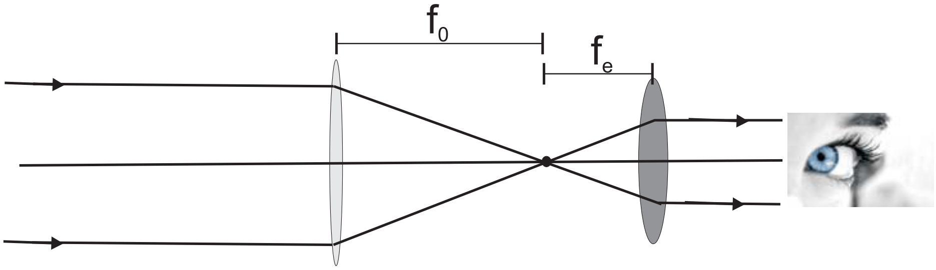


Figure 5: Keplerian telescope

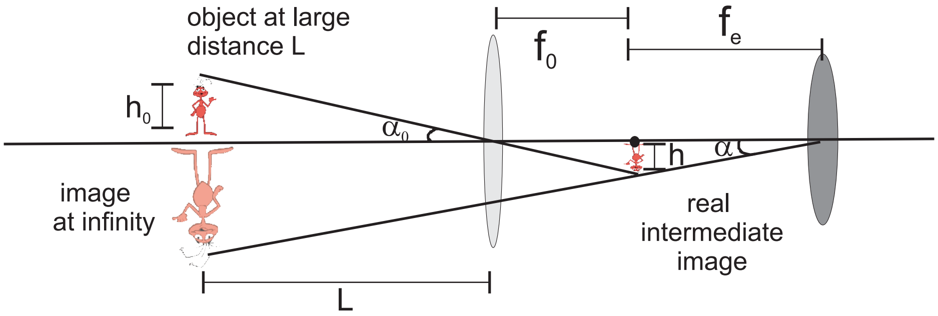


Figure 6: Keplerian telescope2

3 Experimental procedure

In the experimental setup optical components such as lenses, screens and test objects are placed into holders that slide along an optical bench. The optical bench ensures that all the components are aligned correctly.

3.1 Determine the focal length of a converging lens

It is possible to determine the focal length of converging lenses in several ways, and this experiment begins by examining three different methods of measuring the focal lengths of the lenses marked L1 to L3 (which are converging lenses).

- Determine the focal lengths of the lenses by the method of using an object at infinity, i.e. the ceiling light *Remember that for converging lenses, when an object is placed at infinity, its image lies at the focal point with zero size.* Point the lens towards the ceiling light, locate the position of the real image and thereby the focal length.
- Use the displacement method to measure the focal lengths of L1-L3. Place the screen and the object about 1 meter apart, and locate two positions at which the lens makes an image of the object on the screen.
- The object distance and image distance are related by the image equation. Measure the real image distances by lens L1 for 5 different object distances, and find the focal length by plotting a graph of $1/s_i$ versus $1/s_o$.

3.2 Determine the focal length of a diverging lens

We now turn our attention to measuring the focal length of a diverging lens using two different methods: *The displacement method* and *the image shift method*. The schematics of these two methods can be seen in figure 7.

- Displacement method:
  Put the diverging lens at a position 5-15 cm from the lighted object. A glance into the lens shows a virtual image at a position somewhere behind the lens that is smaller than the object. Select L2 as a lens with

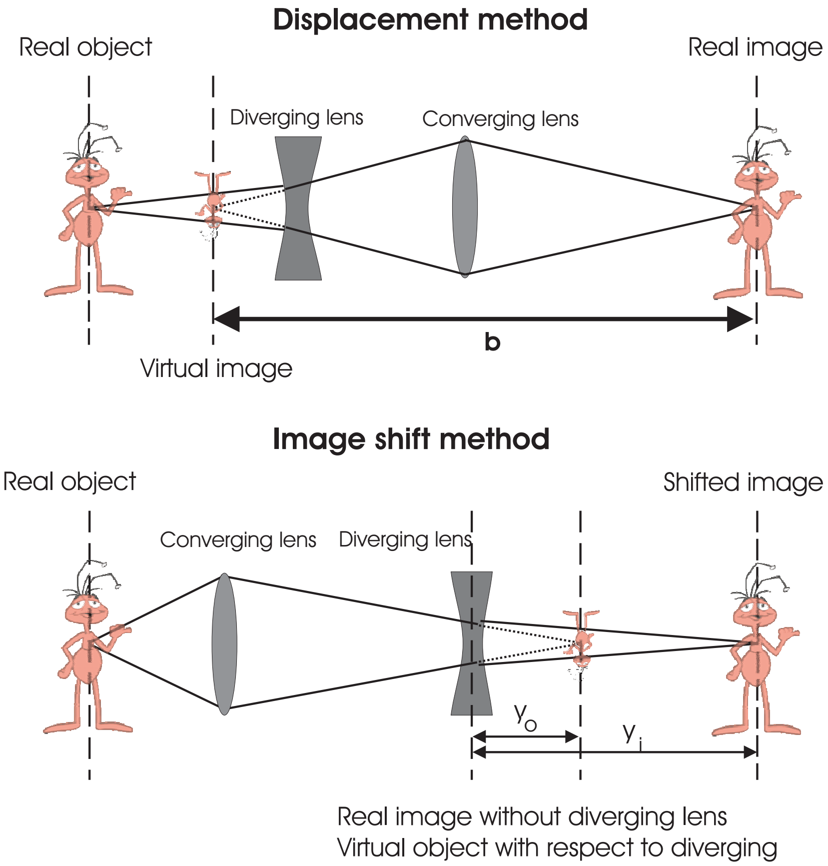

Displacement method

Image shift method

Figure 7: Displacement method and image shift method.

known focal length, f, and place it on the optical bench between the diverging lens and the screen. Use the displacement method to find a value for b. This gives a value for the distance of the virtual image, y_i, from the diverging lens. Equation (1) can then be used to find the focal length of the lens since the object distance, y_o, between the lighted test object and the lens is readily measured.

- Shifting image:
  The focal length of a diverging lens can also be found by measuring how far a real image shifts when a diverging lens is added to the optical system. As shown in Figure 7, suppose a converging lens is used to form a real image on a screen (use L1). When the diverging lens is

placed between the converging lens and the screen, the real image is no longer focused on the screen, but has moved further away. This shift in focal length can be used to obtain a value for the focal length of the diverging lens. When the diverging lens, L4, is inserted the real image disappears. Instead, the lens generates a real image at a new position from the virtual object. Use the positions of the real image and the virtual object to find the focal length of L4.

3.3 Building a simple microscope

Attach the microscope grid to the screen and observe it through lens L2. First bring the lens close to the screen, you will see a slightly magnified virtual image. Then gradually move the lens away from the screen. Keep moving until your eyes can no longer focus on the image; this means that you are close to the focal distance. Try this and verify it. Determine roughly the magnification by counting the number of lines you see with the naked eye that fit in between three or four lines in the magnified image. Compare your finding with that predicted by equation (6).

Construct a compound microscope using the following three step approach.

- Create a real, magnified intermediate image of the illuminated source on the screen using lens L1 as shown in figure 4. Determine the magnification by direct measurement.
- Set up the magnifying glass (L2) to observe the microscope grid on the back side of the screen (just as you did above).
- Replace the illuminated source by the screen with the microscope grid and peer through the microscope. You may have to move the object a bit to adjust the focus. Determine the magnification using the scheme mentioned above and compare with theory.

3.4 Building a Keplerian telescope

Using lenses L1 and L3 to construct a Keplerian telescope with figure 5 as your guide.

Make sure the focal length of L1 and L3 overlap as in figure 5. Tape the telescope grid sheet to the opposite wall and peer through your telescope with one eye. You may have to experiment to find the right lens separations for your telescope. You should see a clear image with a visible magnification when the lenses are properly aligned. Determine the magnification from equation (8). What is the significance of the negative sign in the magnification?

4 To be included in assignment 1

1. The values for the focal length of each lens measured using an object at infinity. (Sec. 3.1)
2. The values of a and b from the displacement method, along with the corresponding focal length. (Sec. 3.1)
3. The experimental plot of the image equation along with the slope and the corresponding focal length. (Sec. 3.1)
4. The focal lengths of the diverging lenses from the displacement and shifting methods. (Sec. 3.2)
5. Derive equation (3) and show that $M_1 M_2 = 1$.
6. The magnification measured for the microscope and its theoretical prediction. (Sec. 3.3)
7. A diagram of your microscope with all positions displayed. Measured and predicted magnifications. (Sec. 3.3)
8. A sketch of your telescope. Show a calculation of the theoretical value for magnification, and state your measured value. (Sec. 3.4)
9. What kind of aberrations have you observed in this experiment?
10. Discuss the sources of errors in the experiment.