The Michelson interferometer

In 1907, Albert Michelson won the noble prize for his work on measuring the speed of light very accurately using a configuration known today as the Michelson interferometer. Originally, Michelson built the interferometer to investigate the existence of the mysterious "ether". It was thought for several decades that the Earth floated in a fluid called ether as it went through its orbit. The results of the famous Michelson-Morley experiment supported the idea that there is no stationary medium through which light propagates, which later formed the basis of Einstein's theory of relatively. Later the interferometer was used to measure the wavelengths of atomic spectral lines with high precision, as well as displacements in terms of wavelengths of light. This enabled scientists to develop high precision length standards as well as improved methods for calibrating length measuring instruments.

1 Purpose of the exercise

- To use and understand the Michelson interferometer.
- To use the interferometer to measure the wavelength of laser light.
- To use the interferometer to measure the index of refraction of air.
- To investigate how changes in pressure affect the index of refraction.

2 Theory of the Michelson interferometer

In this experiment, you will use a Michelson interferometer to determine the wavelength of laser light, as well as to investigate the index of refraction of air and how it is affected by changes in pressure. A diagram of the Michelson interferometer is shown in figure 2. Light from a monochromatic source (a laser) passes through the beam splitter, producing two perpendicular beams of equal intensity. The two beams are then reflected off two separate mirrors, and when they recombine at the beam splitter they will interfere with each other. As you know from the lecture, whether the interference will

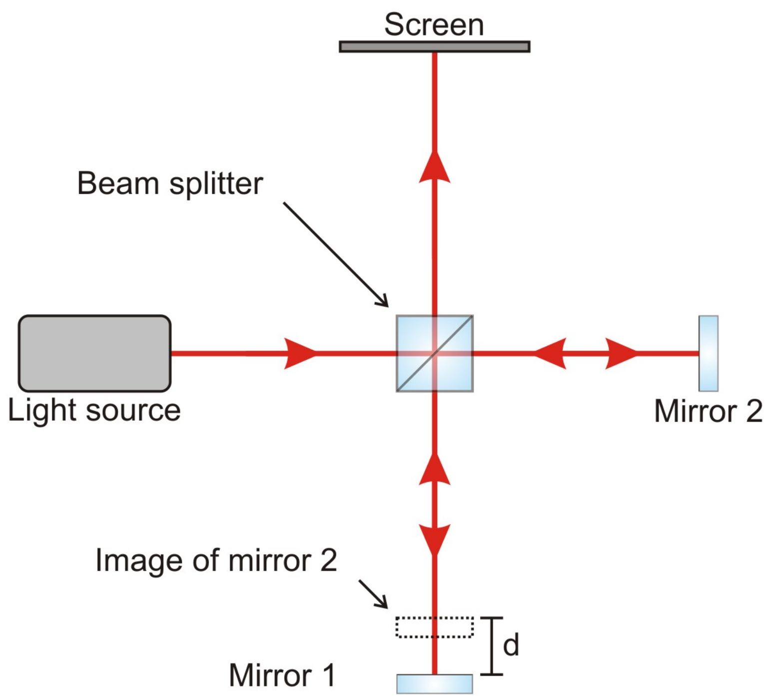

Figure 1:

be constructive or destructive depends on the relative phase of each of the combining beams. This relative phase is determined by the path length difference, $2d$. The condition for constructive interference (the amplitudes add to produce max intensity) is

$$2d = m\lambda \tag{1}$$

where m is an integer and λ is the wavelength of the light. For destructive interference, the recombining beams cancel each other out, and the condition is

$$2d = \left(m + \frac{1}{2}\right)\lambda \tag{2}$$

so when the path length is an odd half integer multiple of the wavelength, the recombining light beams will be exactly out of phase and thus the recombined beam will have zero amplitude provided the amplitudes of the split beams are equal.

By moving one of the mirrors, we can change the path length difference and the relative phase of the light beams. As the path length difference

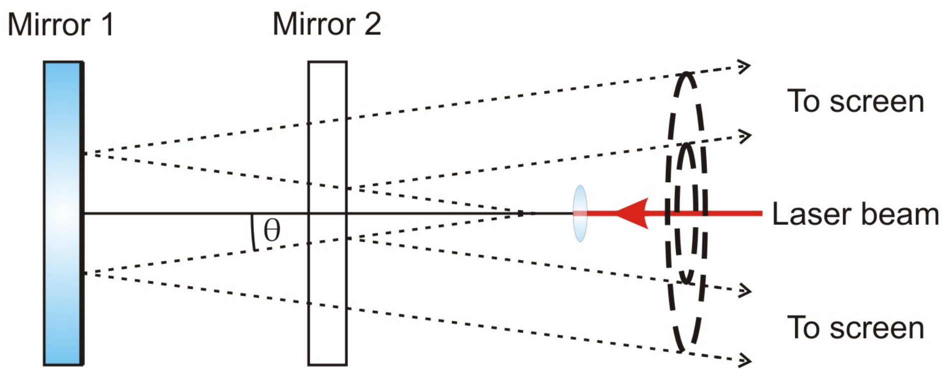

Figure 2:

changes, we would see a continuous change from constructive interference (with maximum brightness) through partial interference (with the spot fading away) to destructive interference (the spot disappears) when the relative distance is increased by one wavelength. In the laboratory you will see this beautiful phenomenon.

For some purposes, like in this exercise, it is more practical to use a dispersed beam instead of a thin collimated laser beam (as the one coming out of the laser source). With a dispersed beam the interferometer produces an interference pattern on the screen instead of a single point, and the principles are shown in figure 2. This is also the kind of interference described in the book on pages 193-196. In figure 2 the primary elements of the interferometer are shown in a linear arrangement. The parallel beams reflected towards the screen interact with each in a constructive or destructive manner, depending on the path length difference. Naturally, the path length difference now depends on the angle, θ, and is given by $2dcos\theta$ similar to (8-1) in the book. Since the path difference is dependent on the angle of the beam there will be certain angles where there is constructive interference and certain angles at which there is destructive interference, thereby producing an interference pattern with bright and dark concentric circles. Just like in the previous case. the condition for constructive interference is

$$2dcos\theta = m\lambda \tag{3}$$

which reduces to (1) for $\theta = 0$ corresponding to the central spot in the interference pattern.

2.1 Measuring the wavelength

When moving one of the mirrors in the interferometer with a dispersed beam, the fringes of the interference pattern will move either outwards or inwards, thus fringes are either annihilated or produced. The central spot of the interference pattern will alternate between bright and dark for every wavelength the path length difference is changed, as discussed above. Therefore, very fine control of the path length difference is required. The path length difference is controlled by attaching a micrometer screw onto one of the interferometer mirrors. The distance traveled by the mirror is equal to the change in the mirror separation, Δd. By counting the number of passing fringes corresponding to a measured mirror displacement, it is possible to calculate the wavelength of laser light. The number of fringes that have passed in a mirror displacement, Δd, is equal to the number of wavelengths that the path length difference has changed. Hence, according to eq. (1) we find

$$\lambda = \frac{2\Delta d}{N} \tag{4}$$

where N is the number of passing fringes corresponding to the mirror separation Δd.

2.2 Understanding the structure of the interference pattern

To understand the origin of the interference pattern we investigate it a bit further. In the exercise you will be asked to measure the fringe spacing of the stationary interference pattern, and then compare it with the theory now introduced. Given a constant path difference and constant wavelength, there are values of θ and m that satisfy eq. (3), which correspond to each fringe. So we have

$$2d\cos\theta_N = m_N\lambda \tag{5}$$

where θ_N is the angle of the N^{th} fringe from the center of the interference pattern, and m_N is the integer number of wavelengths associated with the N^{th} fringe path difference. If we adjust d so that there is a fringe of maximum brightness at $\theta = 0$, then m_0 is the value of m that satisfies eq. (3) for $\theta = 0$. So then $2d = m_0\lambda$ and m_0 is the integer number of wavelengths equal to twice the mirror separation. Since neighboring fringes differ in path length difference by one wavelength, $m_0 - m = N$. Combining all these equations give the following relationship

$$\cos\theta_N = 1 - \frac{N\lambda}{2d} \tag{6}$$

By plotting the cosine of the fringe angle versus the fringe number, we see how well the interference pattern matches the one predicted by the theory.

2.3 Measuring the refractive index

The interference pattern is sensitive to changes in the relative phase of the two split beams. One can therefore use the interferometer to investigate how transparent objects affect the phase of light placed in one of the interferomter arms. In the experiment you will use an air tight cylinder with glass windows on both faces. This air cell will be used to investigate the index of refraction of the air inside.

As a light beam passes through a medium, the wavelength of light is dependent on the index of refraction by the simple formula $\lambda = \lambda_{vac}/n$ where λ is the measured wavelength of the medium, λ_{vac} is the wavelength of the light beam measured in a vacuum, and n is the refractive index of the medium. The number of wavelengths that make up the path length in the air cell, $N_{cell}(p)$ is given by

$$N_{cell}(p) = \frac{2t}{\lambda_{cell}} = \frac{2t}{\lambda_{vac}} n(p) \tag{7}$$

where t is the thickness of the cell, λ_{cell} is the wavelength of the light in the cell, p is the pressure inside the cell, and $n(p)$ is the index of refraction of air at pressure p. Since the index of refraction is a function of pressure, so is the number of wavelengths in the air cell.

Note that the index of refraction of vacuum is 1 and that it is always larger for any other material. This means that the wavelength of a beam of light is maximum in vacuum. Therefore, considering the air cell, the wavelength of the light beam traversing it will increase as the air is evacuated. This means that the fringes of the interference pattern will move as air is evacuated from the cell. The number of passing fringes is equal to the change in the number of wavelengths in the air cell. The number of passing fringes, N_{pass}, at two different pressures, p_{atm} (atmospheric) and p (arbitrary), is

$$N_{pass}(p) = N_{cell}(p_{atm}) - N_{cell}(p) = \frac{2t}{\lambda_{vac}}(n(p_{atm}) - n(p)) \tag{8}$$

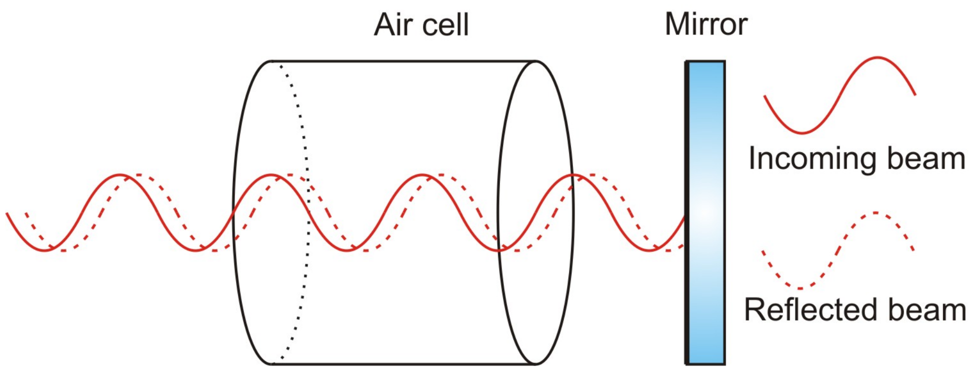

Figure 3:

Rearranging the equation we find

$$1 - \frac{\lambda N_{pass}}{2t} = \frac{n(p)}{n(p_{atm})} \tag{9}$$

where λ is the wavelength of laser light in air at atmospheric pressure. You should use this expression to evaluate the index of refraction of air.

3 Experiment

3.1 (Building and) aligning the interferometer

Begin by aligning the interferometer. The instructor will help you doing this.

3.2 Measure the wavelength of light

You will now use the micrometer to measure the wavelength of the laser light. Turn the micrometer screw slowly counter clockwise while counting the number of passing fringes. After about 80 fringes have passed, record the number of fringes as well as the mirror displacement from the micrometer screw. Repeat the measurement a couple of times.

Make a table of values for the number of passing fringes, N, and the corresponding change in mirror separation, Δd, with errors. Include a column of the calculated wavelength with error.

3.3 The structure of the interference pattern

Adjust the micrometer screw so that the center of the interference pattern is an illuminated dot of maximum brightness. This implies that there is constructive interference at $\theta = 0$ and eq. (1) is satisfied. Measure the radius of each fringe from the center of the interference pattern using the ruler. Record the radius and the corresponding fringe number. Measure the distance from mirror 1 to the wall with a meter stick. From the fringe radius and the distance to the wall, one can determine the angle of the fringe, θ, in order to compare with the predictions made by eq. (6).

Make a table of values for the fringe number, N, with the corresponding fringe radius as measured with errors. From the fringe radius and the wall distance, calculate the $cos\theta$ value for each of the fringes, with errors. For this calculation you can assume the beam is perpendicular to the wall and that the beam does not strike the mirrors far from the center. Plot $(1 - cos\theta)$ versus$N$. Is it a linear relationship with reasonable slope and intercept?

3.4 Measure the refractive index of air

Start by recording the air pressure with a barometer. Place the air cell in the beam path between the beam splitter and mirror 1, and be sure that the air cell is parallel with the beam path. Press the pressure release button on the pump so that the pressure in the air cell is equal to atmospheric pressure. Record the initial pressure reading. Then slowly squeeze the pump until the fringe interference pattern has completed one cycle, and record the pressure again. Repeat this process until the pressure reaches the minimum value and the pump can no longer evacuate air from the cell (when you are finished remember to press the pressure release button).

Make a table of values for the number of passed fringes, $N_{pass}(p)$, and the corresponding pressure, p, as measured and evaluate the possible errors. Include a column of the calculated $n(p)/n(p_{atm})$ values. Then plot $(1 - n(p)/n(p_{atm}))$ versus p with error bars. Extrapolate the plot to find the $(1 - n(p)/n(p_{atm}))$ value at zero pressure. From the intercept, obtain the index of refraction of air at atmospheric pressure.

4 To be included in assignment 2

1. A table of values for the number of passing fringes, N, and the corresponding change in mirror separation, Δd, with errors. Include a column of the calculated wavelength with error, and compare your wavelengths with the expected wavelength (expected is 532nm).

2. A table of values for the fringe number, N, with the corresponding fringe radius as measured with errors. From the fringe radius and the wall distance, calculate the $cos\theta$ value for each of the fringes, with errors. For this calcualtion you can assume the beam is perpendicular to the wall and that the beam does not strike the mirrors far from the center.

3. A plot of $(1 - cos\theta)$ versus N, with error bars. Is it a linear relationship with reasonable slope and intercept? Comment on whether or not the fringe spacing is consistent with equation (6).

4. A table of values for the number of passed fringes, $N_{diff}(p)$, and the corresponding pressure, p, as measured in experiment where the refarctive index of air is to be determined. Include a column of the calculated $n(p)/n(p_{atm})$ values.

5. A plot of $(1 - n(p)/n(p_{atm}))$ versus p with error bars. Extrapolate the plot to find the $1 - n(p)/n(p_{atm})$ value at zero pressure. From the intercept, obtain the index of refraction of air at atmospheric pressure. Compare your measured value of the index of refraction of air with the accepted value of 1.00025.